UG SEMESTER VII

Paper 15: Topology

Credit: 4 T:04

Course Outcomes:

1. Define and illustrate the concept of topological spaces and continuous functions,
2. Illustrate the concept of limit point, dense sets, interior, exterior, boundary points.
3. Identify and understand bases, sub-bases and different type of spaces like Lindelof, Separable, and their properties.

Unit I

Countable and uncountable sets, Schroeder-Bernstein theorem, Cantor's Theorem, Cantor's Sets, Cantor's continuum hypothesis, Zorn Lemma, Well ordering principle.

Topological spaces: Definitions and Examples, open base and open subbase for a topology, Lindelof theorem, limit points, closure, interior; Continuous functions, Homeomorphisms; relative topology, Metric Topology, Product Topology, Weak topology, The function algebras C(X,R) and C(X,C).

Unit II

Compact spaces, Heine Borel theorem, product of spaces, Tychonoff theorem, generalized Heine Borel theorem, locally compact spaces, compactness for metric spaces, Ascoli's theorem.

Unit III

Separation Axioms: T1 and Hausdorff spaces, completely regular and normal spaces, Urysohn's lemma; Tietze extension theorem. Uryohn's imbedding theorem; Stone Cech compactification.

UNIT IV

Connected spaces, the components of a space, totally disconnected space, locally connected space,

References

Text Books:

1. G.F. Simmons: Introduction to Topology and Modern Analysis, Mc-Graw Hill Int. Book Company
2. J. R. Munkres: Topology - A first course, Prentice hall India Pvt. Ltd.

Suggested Readings:

3. J. L. Kelley: General Topology. Van Nostrand. Reinhold Co., New York 1995

Web References:

Digital platforms web links: NPTEL/SWAYAM/ MOOCS/Openstax.org

https://openlearninglibrary.mit/edu/courses

http://heecontent.upsdc.gov.in/SearchContent.aspx

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Paper 16: Fluid Mechanics

Credit: 4 T:04

Course Outcomes:

1. understand the concept of fluid and their classification, models and approaches to study the fluid flow.
2. formulate mass and momentum conservation principle and obtain solution for non viscous flow.
3. know potential theorems, minimum energy theorem and circulation theorem.
4. understand two dimensional motion, circle theorem and Blasius theorem.

Unit I

Types of fluids, Continuum hypothesis, Lagrangian and Eulerian method of describing fluid motion, Motion of Fluid element: Translation, Rotation and Deformation. Stream lines, Path lines and streak lines. Material derivative. Acceleration of a fluid particle in Cartesian, Cylindrical Polar and Spherical Polar Coordinates. Vorticity Vector, Vortex Lines, Rotational and Irrotational motion of fluid, Rotational velocity, Velocity Potential, Boundary surface, Boundary condition.

Unit II

Reynold transport theorem. Principle of conservation of Mass-Equation of continuity (By Lagrangian and Eulerian method). Equation of Continuity in different coordinate systems. Body force and Surface force. Euler's equation of motion-conservation of momentum, Bernoulli's Equation, Energy Equation, Impulsive effects.

Unit III

Irrotational motion in two dimensions: Stream function, Physical significance of stream function, Sinks, Doublets and their images in two dimension, Complex Velocity Potential. Sources, Milne-Thompson circle theorem, Vortex,

Vortex motion, Image of Vortex, Kelvin Circulation Theorem, Complex potential due to Vortex, Kirchhoff vortex Theorem, Blasius Theorem and Kutta-Joukowski Theorem.

Unit IV

Irrotational motion produced by motion of circular cylinders in an infinite mass of liquid, Liquid Streaming past circular cylinder, Kinetic energy of liquid, Motion of sphere through a liquid at rest at infinity. Liquid streaming past a fixed sphere, Axis-Symmetric flow, Stoke's function.

References:

Text Books:

1. Frank Chorlton: Text Book of Fluid Dynamics, C.B.S. Publishers, Delhi.
2. Z.U.A. Warsi: Fluid Dynamics, Theoretical and Computational Approaches, C.R.C. Press
3. S.W. Yuan: Foundation of Fluid Mechanics, Prentice Hall of India Pvt. Ltd. New Delhi
4. N. Curle and H J Davies: Modern fluid dynamics

Suggested Readings:

5. G. K. Bachelor: An Introduction to Fluid Dynamics. Cambridge University Press. London.
6. R.W. Fox, P.J. Pritchard and A.T. McDonald: Introduction to Fluid Mechanics, Seventh Edition, John Wiley & Sons, 2009.

Web References:

Digital platforms web links: NPTEL/SWAYAM/ MOOCS/Openstax.org

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Paper 17: Differential Geometry of Manifolds

Credit: 4 T:04

Course Outcomes:

1. Elaborate the concept of differentiable manifolds and their examples.
2. Clarify the concepts of vector fields, tangent vectors & tangent spaces in a manifold.
3. Apply various concepts of differential calculus to the settings of abstract set called manifold.

4. Use Riemannian metric on a given manifold to find the various types of curvatures with emphasis on the surface/ types of manifold.
5. Bring out different connections on Riemannian manifold and its properties.
6. Calculate curvature tensor & tensors of respective connections.

Unit I

Definition and examples of differentiable manifolds, Tangent vectors, Tangent Spaces, Vector fields and their examples, Jacobian map. Immersions and submersions, Diffeomorphism and their examples, Curve in a manifold, Integral curves and their examples, Distributions, Hypersurface of R^n, Sub-manifolds.

Unit II

Standard connection on R^n, Covariant derivative, Sphere map, Weerigarten map, Gauss equation, the Gauss curvature equation and Coddazi-Mainardi equations.

Unit III

Invariant viewpoint, Cartan view point, coordinate view point, Difference Tensor of two connections, Torsion and curvature tensors.

Unit IV

Riemannian Manifolds, Length and distance in Riemannian manifolds, Riemanian connection and curvature, Curves in Riemannian manifolds, Submanifolds of Riemannian manifolds.

References:

Text Books:

1. N.J. Hicks: Notes on Differential Geometry, D. Van Nostrand, 1965.
2. U. C. De., A. A. Shaikh: Differential Geometry of Manifolds, Narosa Publishing House.

Suggested Readings:

3. Y. Matsushima: Differentiable Manifolds, Marcel Dekker, INC. New York, 1972.

Web References:

Digital platforms web links: NPTEL/SWAYAM/ MOOCS/Openstax.org

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Paper 18: Complex Analysis

Credit: 4 T:04

Course Outcomes:

1. Understand the topics of Complex Analysis needed to pursue research in pure mathematics.
2. Understand the properties of maximum modulus of a Complex valued function and the results based on that property.
3. Develop manipulation skills in the use of Rouche's theorem and Argument Principle.
4. Show knowledge of Gamma and Zeta functions with their properties and relationships.
5. Understand the Harmonic functions defined on a disc and concerned results.
6. Make factorization of entire functions having infinite number of zeros.

Unit I

Maximum Modulus Theorem, Schwarz's Lemma, Minimum Modulus Theorem, Hadamard's three circle theorem, automorphism of the unit disk. Convergence of sequences and series of complex numbers, absolute convergence. Uniform convergence of sequence and series of functions, Cauchy's criterion, Weierstrass's M-test, analytic convergence theorem. Absolute and uniform convergence of power series, integration and differentiation of power series, radius of convergence.

Unit II

Zeroes of holomorphic functions, Open Mapping Theorem, Inverse Function Theorem. Index of a closed path, meromorphic functions, argument principle, Rouche's theorem, residueat the point at infinity, indentation around a branch point and the branch cut, summation of series.

Unit III

Function spaces: Hurwitz theorem, Infinite products, Weierstrass factorization theorem, Mittag-Leffler's theorem, Gamma functions and its properties, Riemann's Zeta function.

UNIT IV

Uniqueness of direct analytic continuation, Power series method of analytic continuation, Natural boundary, Schwarz's reflection principle, Harmonic Functions, Mean value property for harmonic functions, Harnack's inequality, Poisson formula, Jensen's formula, Poisson-Jensen's formula, Convex functions, Hadamard's three circle theorem as a convexity theorem, Canonical products, Hadamard's factorization theorem, order of entire functions.

References:

Text Books:

1. J. V. Deshpande: Complex Analysis, Tata McGraw-Hill Publishing Company Limited, New Delhi
2. E. C. Titchmarsh: Theory of functions, Oxford University Press
3. John B. Conway: Functions of one complex variables, Springer International

Suggested Readings:

4. R.V. Churchill, J.W. Brown, Complex Variables and Applications, McGraw Hill.

Web References:

Digital platforms web links: NPTEL/SWAYAM/ MOOCS/Openstax.org

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http://heecontent.upsdc.gov.in/SearchContent.aspx

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Paper 19 A: Module Theory

Credit: 4 T:04

Course Outcomes:

1. Identify cyclic modules, simple modules, finitely generated modules etc.
2. Find a basis of a free module.
3. Use the basis to describe module homomorphisms.
4. Describe the structure of a finitely generated module over a PID.

Unit I

Modules-Definition and examples, simple modules, submodules, Module Homomorphisms, Quotient modules, torsion free and torsion modules, direct sum of modules.

Unit II

Exact sequences, Short exact sequence, split exact sequences, Five-lemma, free modules, modules over division rings are free modules, invariant rank property.

Unit III

Free modules over PID's, Invariant factor theorem for submodules, Finitely generated modules over PID, Chain of invariant ideals, Fundamental structure theorem for finitely generated module over a PID,

Unit IV

Projective and injective modules, Divisible group.

References:

Text Books:

1. V.Sahai and V. Bist: Algebra, Fourth Edition, Narosa.
2. I.B.S. Passi and I.S. Luther: Algebra, Volume 3 Modules, Narosa

Suggested Readings:

3. S. Lang: Algebra, Addison Wesley.

Web References:

Digital platforms web links: NPTEL/SWAYAM/ MOOCS/Openstax.org

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Paper 19 B: Measure Theory & Integration

Credit: 4 T:04

Course Outcomes:

1. Display understanding of the essential foundations of important aspect of mathematical analysis.
2. Explain the measurability of a set of real numbers and measurable functions.
3. Differentiate between the Riemann integral and the Lebesgue integral.
4. Apply the Measure theory and theory of the integral in other branches of pure and applied mathematics.

Unit I

Algebra of sets, countable sets, Cantor set, Borel sets, outer measure of a set and its properties. Measurable sets. Lebesgue measure, a non-measurable set.

Measurable functions and their properties. Concept of almost everywhere. Littlewood's three principles.

Unit II

The Lebesgue integration of bounded function over a set of finite measure, the Lebesgue, Bounded convergence theorem, the integral of a non-negative function, Fatou's Lemma, Monotone convergence theorem, the general Lebesgue integral, Lebesgue convergence theorem.

Unit III

Differentiation of monotone functions, Vitali's Lemma, the four derivatives, the differentiation theorem. Functions of bounded variation, Differentiation of an integral. Absolute continuity.

Unit IV

Inequalities and the Lp Spaces: The Lp Spaces, convex functions, Jensen's inequality, the inequalities of Holder and Minkowski, completeness of $L_p(\mu)$. Convergence inmeasure, almost uniform convergence.

References:

Text Books:

1. H.L. Royden: Real Analysis, Pearson Prentice Hall
2. G.de Barra: Measure Theory and Integration, Wiley Eastern Ltd.

Suggested Readings:

3. Taylor, Measure Theory and Integration,American Mathematical Soc., 2006

Web References:

Digital platforms web links: NPTEL/SWAYAM/ MOOCS/Openstax.org

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Research Methodology

Credit: 4 T:04

Course outcomes:

This course is designed to enable students to:

1. Identify and discuss the role and importance of research.
2. Identify and discuss the issues and concepts salient to the research process.
3. Identify and discuss the complex issues inherent in selecting a research problem, selecting an appropriate research design, and implementing a research project.
4. Identify and discuss the concepts and procedures of sampling, data collection, analysis and reporting
5. Read, comprehend and explain research article and writing a research article.

UNIT I

Research Formulation and Design

Motivation and objectives – Research methods vs. Methodology. Steps of research, Types of research – Descriptive vs. Analytical, Applied vs. Fundamental, Quantitative vs. Qualitative, Conceptual vs. Empirical, concept of applied and basic research process, criteria of good research. Defining and formulating the research problem, selecting the problem, necessity of defining the problem, importance of literature review in defining a problem, literature review-primary and secondary sources, reviews, monograph, patents, research databases, web as a source, searching the web.

UNIT II

Data measurement and Data Analysis

Measurement: Concept of measurement, Problems in measurement in research – Validity and Reliability. Levels of measurement – Nominal, Ordinal, Interval, Ratio. Observation and collection of data, methods of data collection, sampling methods, data processing and analysis strategies and tools, data analysis, hypothesis testing.

UNIT III

Soft Computing

Computer and its role in research, some mathematical software like MATLAB, R etc. and their application in research. Software for paper formatting like LaTeX/MS Office.

UNIT IV

Research Ethics and Report Writing

Significance of Report Writing, Different Steps in Writing Report, Layout of the Research Report, Ethical issues related to publishing, Plagiarism, Software for detection of Plagiarism, publishing a research article.

REFERENCES:

1. Kothari, C.R., 1990. Research Methodology: Methods and Techniques. New Age International.
2. Garg, B.L., Karadia, R., Agarwal, F. and Agarwal, U.K., 2002. An introduction to Research Methodology, RBSA Publishers

Suggested Readings:

3. A Manual for Writers of Research Papers, Theses, by Kate L. Turabian, Wayne C Booth, Gregory G. Colomb.

Web references:

https://www.classcentral.com/course/swayam-research-methodology-17760
http://users.cla.umn.edu/~nwaller/math.htm